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THE COMPREHENSIVE EXAMINATION IN MATHEMATICS.

By Ernest H. Koch, Jr.

My topic, like every other educational problem, is an old one in a new dress. We would make no progress in educational matters if it were not for the fact that our educational systems are undergoing readjustments in accordance with the demands of life.

Three thousand years ago China examined her candidates for public service by subjecting them to a systematic test. Since that time we have continued to examine candidates for service, promotion and honors by subjecting them to various kinds of examinations which are typefied by the adjective modifiers, superficial, brief, thorough, protracted and searching. We must add one more, the comprehensive examination, which represents the very last word in educational thought.

The elements of our examinations are traceable to distinct periods in the history of education. The method of conducting examinations in oral or written form, in practical test and by thesis or any combination of these is a matter of expediency.

A *comprehensive examination* is a rational attempt to test the degree of adjustment of an individual in the scheme of life. Such an examination should reflect the sociological and psychological aims of education, and then their purpose, necessity and utility will become comprehensive in name as well as in facts, not only to the teachers but also to the greater bodies of students and laymen. As soon as we take a more sympathetic attitude with the realities of life then we appreciate that success in life depends not only upon capacity and strength but also upon the fitness of the individual. You will agree with me then that what we should do in our examinations is not only to test for range and depth of knowledge but also to ascertain the individual's powers of co-ordination, reflection, expression, commonsense, skill in manipulation, application, and higher powers of investigation. We are meeting daily the inquiry on the efficiency of our school work by reiterating little set speeches. At the same time we are deceiving ourselves, because we know that the majority of examinations set false standards which not only negate good teaching but retard the stimuli which nature intended should be utilized to promote effort, scholarship, character and activity in the larger problems of citizenship. The comprehensive examination is the first instrument that we have forged by means of which we hope to achieve some real success in this business of education. It suggests the means for making our training efficient and in deciding the important factors in education and their limitations. We must gain and then maintain a sympathy with the problems of life so that the transition from the school world to the larger environment may require a minimum readjustment. The power of comprehensive vision is what we cherish. The knowledge which we have imparted and the character training which has been developed is of no consequence unless there is back of it the power to discern the true relation of things. Would you expect a great electric company to install an expensive hydro-electric plant on the banks of a mighty stream and then leave it inoperative without any directing force to make a judicious distribution of the available power among the many related industries which it has promoted and made dependent upon it?

Our examinations have not been comprehensive because we have failed to take a comprehensive view of knowledge. We have been content to regard the facts of history, science, politics, religion and literature as if they were distinct entities and therefore unrelated. We have failed therefore because we have neglected to follow five fundamental considerations:

1. The child is entitled to his scientific inheritance, to his literary inheritance, to his æsthetic inheritance, to his institutional inheritance and to his religious inheritance. This body of experiences is of no value to the individual unless he can discern the general law of truth which binds and permeates them. What is this symbol of truth from which all the fabrics of knowledge are woven?

2. The true purpose of examinations is to indicate the quality and measure the quantity of individual adjustment to an arbitrary standard determined by the aim of education. This implies the elimination of conflicting aspects of education as interpreted by primary, secondary and collegiate institutions.

3. "All knowledge is either envisaged or implied in data of experiences, and it is man's reasoning faculties which make explicit what is thus implied. Man is the measure of all things."

4. The efficiency of any examination system unless wisely administered by a philosopher is subject to the frailties of human judgment. We must be mindful of the paradox which states that every individual is a triplicity of identities. He is what he seems to be to others, he is what he thinks he is and he is what he is.

5. The form and formalism of an examination should always be free from the vagaries of personal idiosyncracies. The place of examinations in the educational system, their duration, their gradation and their flexibility should depend upon the exigencies of the occasion. The complete or partial substitution of a certificate or its equivalent is a recognition of the efficiency of the institution's internal periodic examinations.

These remarks may suffice to introduce us to the more special topic of the comprehensive examination in mathematics. I hope my desire to be brief has not made my introduction seem fragmentary nor so comprehensive as to be extralimitary to our experiences.

Is mathematics justified in our curriculum and if so what are its sociological and psychological values? The answer to this question must state: (1) the meaning of mathematics; (2) the aims of mathematics teaching; (3) the relation of mathematics to other departments of knowledge; (4) the meaning of a comprehensive examination in mathematics; (5) the reaction of rational mathematics upon the comprehensive examination.

The educational value of mathematics is both utilitarian and cultural. The metrical characteristics of mathematics contribute largely to the sociological aspects, whereas the non-metrical characteristics of mathematics contribute largely in the psychological aspects of the question.

Number, one of the gifts of language, enters daily into our lives because it expresses rank, order, greatness, vastness, extent, size, intensity, activity, strength and significance of quantities. Form enters likewise into our daily lives because it limits and visually defines the magnitudes of our experiences. Therefore number and form serve as symbols to express and define the world of relativity of which we are a part. Therefore mathematics is justified from the sociological aspect of education.

Mathematics promotes accuracy of thought, logical relationships between cause and effect in various kinds of reasoning processes. It gives a means for the interpretation of phenomena and directs the mind to an appreciation of unobserved conditions within and without the realm of the data of experience. Therefore it promotes judgment, imagination and orderly habits of thinking. In applied work it promotes fidelity, poise, commonsense and tact. Therefore mathematics is justified from the psychological aspect of education.

The cardinal virtue of mathematics lies in the fact that in its vast symbolism it is the essence of truth. It is the one great potentiality which underlies and permeates all matter, force and thought. Its non-metrical phase is a measure of the human recognition which has been given to its essential omnipresence.

The meaning of mathematics is relative and varies from the extreme opinion of the biased non-mathematician to the ecstatic feeling of the genius in pure mathematics. Mathematics may be valued as a mere tool, an art of computation or as a transcendent mirror capable of projecting the infinite into the domain of the commonplace and capable of reflecting and revealing the commonplace so as to expose its subtleties and unsuspected beauty.

The non-mathematician regards mathematics as inert and stupefying in the mere majesty of its magnitudes. The mathematician energizes matter with a living reality, gives philology the promise of the fulfillment of its myths, assures the ideals of literature, art and history, endows biologic and physical sciences with an impetus that lifts them out of a sphere of mere nomenclature, interprets for religion the meaning of ancient lore and ecclesiastic doctrine, vitalizes the statistics of social forces and in every sphere of intellectual activity it enters as law and emerges as harmony and beauty. Thus it may be said of mathematics that it is justified from its many-sided purpose for it does enter all the spiritual possessions of the race.

I believe that some of our mathematicians have become so engrossed with their specialty that they have sacrificed their sympathy with the masses. It is this type of specialist who is a self-styled authority and who compiles a mathematics syllabus so that the whole range of subjects from arithmetic to calculus shall serve no other purpose than to lend support to each other in their apparently unalterable sequence. The baneful influence of such reports is regrettable because such a committee sets itself up as an infallible model and then it requires untiring effort during the decade following to rectify the diverted stream of progressive thought. In sharp contradistinction to such reports as referred to are the extremely valuable reports of the international committee on the teaching of mathematics which are being published by courtesy of the various governments.

If we were convinced that the comprehensive examination would promote the teaching of mathematics we would still find one very important impediment which would have to be removed. I refer to the traditional resentment which one specialist shows toward another specialist on account of the imaginary encroachments and overlapping of the artificial boundaries of their departmental subjects. The time has arrived when these arbitrary divisions of knowledge must be reconstructed so that they may be interlocking and interdependent. It seems to me that the true function of a principal or director of a school will be restored and he will no longer serve as a highly paid clerk if he will, from time to time, unify the several activities of the classroom and test their contributary strength by applying comprehensive examinations of the most general kind. We need information about our problem and we need it very badly, but we do not want mere criticism because the only kind which is of any value is constructive criticism which indicates breadth of view and is representative of the best thought of the world.

There is one other point which should receive our attention because it has a very important bearing upon the unification of the subject matter as now taught in the schools. We have been so conceited in the past as to believe that mathematics is the only subject which could furnish a suitable backbone, or tree, if you like the simile better, upon which to hang all the other subjects. You will recall that Franklin warned his compatriots that if they did not hang together they would hang separately. The same aphorism applies to the artificial divisions of knowledge and explains the loss of Greek and the threatened loss of Latin in the high schools.

I believe that as mathematics teachers we shall have to concede that any subject in the present division of knowledge may be used as the tree upon which to support the other branches of knowledge. If we take this point of view then the comprehensive examination may be likened to one of the great arteries of travel which links us into a great brotherhood and which distributes equitably the gifts of knowledge which the Great Father has bestowed upon us as a divine heritage. Undoubtedly there will be a readjustment of our departments of knowledge in conformity to the spirit of the age. The tendency in education continues democratic in spirit and the growth of knowledge in the individual is not unlike the development of knowledge historically. Knowledge begins with a small nucleus of facts. With the subsequent acquisition of more facts and their interrelation there arises a differentiation and with further growth we have the beginning of the larger divisions of knowledge. Consequent growth means specialization and further subdivision of departmental knowledge. At this stage an overlapping process brings all knowledge into common relation and the whole mass of knowledge is refined, reassembled and reorganized into new departments and the process repeated indefinitely. In view of these facts the comprehensive examination must be at all times a beacon to our progress. One of the advances which has been made in recent years is the presentation of mathematics in one of the forms known as applied, or practical or vocational mathematics. At the present time there are many high school teachers who are preparing corresponding mathematics texts which it is hoped may prove more adaptable to the needs of the recently established comprehensive high school courses. After all has been said and after all the careful pruning has been done we may hope to have mathematics pure and simple, applied and unadulterated. I am willing to admit that a beginning along these lines antedates the present period of discussion and much credit must be given to such typical organizations as the college entrance boards, but they have much to learn both in the setting of the questions as well as in the more philosophic grading of the answers.

In the setting of a comprehensive examination in mathematics the method may be analytic or synthetic. In the analytic method we collect a group of sterilized questions from which we select a number with special characteristics. In the synthetic method we construct questions so they may possess special characteristics. Such characteristics much emphasize the qualities which we have stimulated in the student. These characteristics will depend upon the kind of students with whom we are dealing and also upon their teleologic training. In a terminal examination it is inexpedient to require an answer which is unrelated to the student's experience and environment. The examination should be graded according to the mental and not the physical age of the student. I believe that the psychologic study of errors will show that the radical defect of mathematics teaching is due to the artificial forcing of latent powers before their natural period of functioning. In consequence of this unnatural procedure we develop a condition of mathematical weakness or even mathematical sterility. If the type of questions are such as would require the use of a book of reference ordinarily, then most assuredly should we permit the use of a reference book during the examination involving that type of question.

There are various categories of qualities, capacities, actions and contents of the mind which we must set down as desirable considerations in preparing and grading an examination. It is evident that these considerations will compel us to resort to some scientific classifications into groups. The following tentative grouping may assist us in estimating the value of these considerations:

Group (*a*) contains the persistent considerations such as accuracy, facility, memorizing number relations, skill in observation, definiteness in information, insight, imagination, reason and judgment.

Group (*b*) contains immediate considerations such as the ability to marshal facts within the subject or in a subject immediately related thereto, clearness of conception in the grasp of principles and in the comprehensive mastery of the subject.

It is not my purpose to outline a definite mode of procedure for the presentation of an examination because I believe that the form of an examination must be determined by the very nature of the qualities or abilities which we may wish to examine. It may be that a brief oral examination will suffice to form a proper basis of judgment and again it may require a long range of short periodic tests, culminating in a certificate which should set forth in a specific statement the characteristics and abilities of the student. Such a certificate should be supplemented by either a brief interview or a written examination or both, in a formal or informal manner and attested by a careful inspection of the student's work books, and other school products of industry.

The supreme test of our educational labors and of the student's achievements and status may be summarized in the question: Is the student fitted to do the work which he next undertakes? I have endeavored to collect from many sources the qualities which mathematics instruction is supposed to foster and accordingly I have enumerated them below.

(*A*) Command of Resources:

1. Capacity to absorb knowledge.
2. Capacity to reproduce knowledge.
3. Love for knowledge.
4. Self-knowledge.
5. Self-control.
6. Self-respect for virility.
7. Readiness for occasion.
8. Spontaneity.
9. Persistence.
10. Patience.
11. Willingness to submit one's self or one's work to scrutiny.

(*B*) Comprehensive Vision:

1. Breadth of knowledge.
2. Clearness of conception.
3. Ability to co-ordinate.
4. Ability to generalize.
5. Ability to visualize.
6. Breadth of appreciation.
7. Imagination.

(*C*) Critical Acumen:

1. Precise discrimination.
2. Quantitative discrimination.

3. Qualitative discrimination.
4. Judgment.
5. Poise.
6. Initiative.
7. Interpretation.
8. Ability to reason independently.
9. Ability to conduct investigation by analysis.
10. Ability to conduct investigation by synthesis.

(*D*) Constructive Articulation:

1. Ability to marshal facts.
2. Ability to transmit intelligence.
3. Ability to express facts concretely.
4. Ability to originate.
5. Accuracy.
6. Skill.
7. Facility.
8. Mastery of subject.
9. Respect for mental effort.
10. Efficiency in quantitative and qualitative performance.

The list which is herewith appended is intended as a suggestion for a more scientific classification. No attempt is made to define the terms and therefore they are to be interpreted with a flexibility that will make them adjustable to the progressive moods of our civilization. At this point it is well to note that these pretentious claims are subject to the limitations of transferability to other subjects in the light of the criticism of the doctrine of formal discipline.

Before you and I can determine the application of the comprehensive examination upon the student we must subject ourselves to the most rigid comprehensive self-examination on the validity of our claims. In setting a comprehensive examination let us draw upon any of the subject matter which has come within the experience of the student. Then we must set down in some order the topics under the groups (*A*), (*B*), (*C*), (*D*). We should then decide upon the grading of each question and then frame a satisfactory answer which would receive full credit. Then the examination should be put away in cold storage and the questions and the model answers should be tested and weighed for their merit. If they are satisfactory after this process of inspection try them.

Comprehensive examination for class . . . range of subject matter. . . . Arithmetic, algebra, plane geometry.

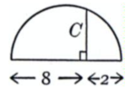
Test 8–*D*7 for speed, time 3 minutes.

Write the answers to the following questions:

1. $\frac{\frac{8}{6}}{\frac{2}{3}}$.

2. $\frac{3x}{2} = \frac{5}{8}$.

3. $c = ?$



The qualities displayed are:
Command of resources, *A* 7.
Comprehensive vision, *B* 1, 6.
Critical acumen, *C* 2, 7.
Constructive articulation, *D* 5, 7.

Test 8–*D*5 for accuracy, time 1¼ hrs.

Show the work in full, make extra computations, if necessary, near the right margin and draw a line through such scratch work, indicate the answer and put the work in good form.

1. Temperature Fahrenheit in terms of Centigrade:
F° = temperature in Fahrenheit degrees.
C° = temperature in Centigrade degrees.
$F^\circ = \frac{9C^\circ}{5} + 32$; solve for C when $F = 70^\circ$.

2. Construct a line *AG* three inches in length, divide it into six equal segments marking the points of division in alphabetic order. With *D* as center construct a circle I with radius $= AD = DG$. With *D* as center construct a circle II with radius $= BD = DF$. With *C* as center construct a circle III with radius $= AC = CE$. (*a*) Determine the area included between circles I and II; (*b*) determine the area included be-

tween circles I and III; (*c*) determine the area common to the circles II and III.

3. Determine the altitude constructed upon the hypotenuse of a 30–60 right triangle when the short leg equals 2.86″. Express the answer with three significant figures.

4. Compute the square of 19.5 by considering it $\left(20 - \frac{1}{2}\right)^2$.

5. Two paper hat boxes have square bottoms. What is the ratio of the areas of their bottoms when their edges are in the ratio of 6:7?

6. Determine the locus of points equidistant from the sides of a parallelogram.

The qualities displayed are:

Command of resources, *A* 2, 3.
Comprehensive vision, *B* 2, 3, 5, 7.
Critical acumen, *C* 2, 3, 4, 6, 7, 9.
Constructive articulation, *D* 3, 5, 6, 8, 10.

Test 8–*A*1 for abstract work, time 1 hr.

1. Solve $x - \sqrt{x^2 - 17} = \frac{4}{\sqrt{x^2 - 17}}$

2. Prove: the diagonals of a parallelogram bisect each other.

3. Draw a parallel to the base of a triangle such that its length between the legs of the triangle shall equal the greater segment of either leg.

4. Factor $(x - y - z)^3 - x^3 - y^3 - z^3$.

The qualities displayed are:

Command of resources, *A* 1, 2.
Comprehensive vision, *B* 1, 2, 4, 7.
Critical acumen, *C* 6, 7, 9, 10.
Constructive articulation, *D* 2, 4, 7.

Test 8–*B*3 for co-ordination.

This test should be conducted so that the student has the privilege to consult any book of reference.

1. (*a*) Make a diagram of the room in which you are being examined; (*b*) estimate its length, breadth and height, dimension the diagram; (*c*) from these figures determine the volume of the room; (*d*) make diagrams of the walls which have windows admitting light from the outside, dimension the diagrams estimating the window openings; (*e*) determine the percentage of each wall area, which is used for lighting the room from the exterior; (*f*) estimate the angle at which the light enters them by observing shadows; (*g*) determine the volume of air which must be admitted to the room per hour if each individual requires 25 cu. ft. per min.; (*h*) determine the length of the diagonals of each wall and also of the floor; (*i*) determine the diagonal of the room; (*j*) estimate the weight of the individuals and the furniture upon the floor; (*k*) show how you would estimate the average age of the individuals in the room; (*l*) determine the total age of the individuals in the room from your individual estimate. If this number is used as a date what historic fact would it bring to your mind? (*m*) What kind of work or study do you plan to do next and how is your choice influenced by your knowledge of mathematics.

The qualities displayed are:

Command of resources, *A* 1, 3, 4, 8.
Comprehensive vision, *B* 1, 2, 3, 4, 5, 6, 7.
Critical acumen, *C* 3, 4, 10.
Constructive articulation, *D* 2, 3, 9, 10.

The tests which are outlined above take on a deeper significance according to the maturity of the student but it is evident that a more careful study of the comprehensive examination will show that they may be arranged to reveal a greater range of student character. There should be some supplementary data which represents the student's own reply to typical question blanks ascertained at the time of his entrance to the school. These personality reports should bear upon his experience, previous training and habits. These should be supplemented by an interviewer's report which is a school officer's estimate of the candidate made at the time of his candidacy. Such an interviewer's blank notes the following facts about the candidate at the time of his interview or oral examination: Name; address; date of birth; age; weight; height; appearance (pleasing, neat, or otherwise); nervous state (quiet, restless or otherwise); manner (attractive, gentlemanly, business-like or otherwise); maturity; intelligence; health; finances; common sense; seriousness; desirability; recommendations of interviewer; date; name of interviewer.

Entrance Information Sheet.

1. Date ....Course applied for ....
2. Name in full ....
3. Home address ....
4. Height ....Weight .... Age: Years ....Months......
5. Did you graduate from grammar school?........................
6. How many years did you spend in high school? ...Did you graduate?..
7. When did you leave the high school? ....
8. What other schools have you attended? ....
9. How long in each? ....Did you graduate?..............
10. How many months have you studied algebra?......Geometry?......
11. Physics? ....Chemistry? ....Mechanical drawing?........
12. Where do you expect to live if you enter the Institute? ....
13. Have you ever before taken the entrance examinations at..........
    Institute? ....
14. Occupation of father ....
15. By whom recommended to..........Institute
    Name ....
    Address ....
16. Is the person who recommended you a..........graduate?.........
17. By what company is he employed?................................
18. By what companies have you been employed? (1).................
    (2) .... (3) ....
19. How many months with each? (1).........(2)........(3)........
20. What was the exact nature of your work with each? (1)..........
    (2) .... (3) ....
21. What were your maximum wages? ....

Entrance Information Sheet.

No application will be considered unless all information called for is given.

19. Do you have enough money to pay the expenses of the two years?....
20. Have you saved it from your earnings or will it be furnished?......
    ....
21. What kind of work do you prefer?...............................
22. Have you ever had any serious illness?.............What..........
23. Are you in vigorous health now?.................................
24. Are the following normal: sight?......hearing?......smell?......
25. Can you arrange to live within not more than one hour's travelling distance from the Institute?..................................

26. In case not admitted: (1) Can you return to the high school for another year? ....
(2) Can you get a position for a year before again asking admission? ....
27. Why do you wish to take the course selected?....................
28. If it is impossible to admit you to the course you desire, what is your second choice? ....
29. If admitted, do you think you have sufficient earnestness and perseverance to pursue your work uninterruptedly and to devote from three to four hours each evening to the preparation of the work for the following day? ....
....

ENTRANCE INFORMATION SHEET

34. Address ....Birthplace ....
35. Where has most of your life been spent? ....
36. Are both parents living? ....
37. Father's occupation ....
38. To what extent are you interested in machinery, sports, athletics?
....
39. Do you play any musical instrument? ....
40. What books have you read? ....
41. What technical periodicals do you read? ....
42. What study do you prefer? ....
43. What are your religious preferences? ....
44. To what extent do you use tobacco and in what forms? ....
45. Are you strictly temperate? ....
46. On the back of this sheet describe in detail any shop, business, or other practical experience which you have had. State also what tools you can use and give an account of anything you have built.

INTERVIEWER'S SUMMARY.

Name of candidate ....Course ....
Address ....
1. Date of birth ....Age ....Weight .... Height........
2. Appearance: pleasing, neat, or otherwise ....
Manner: attractive, gentlemanly, businesslike or otherwise ....
....
3. Nervous state: quiet, restless or otherwise ....
4. Maturity ....
5. Intelligence ....
6. Health ....
7. Finances ....
8. Common sense ....
9. Seriousness ....
10. Second choice ....

11. Desirability ....
12. Recommendations ....
....
....
....
....
....
....

Signed:

Date:

At the present moment New York City and I dare say many other communities are in the midst of a tremendous educational upheaval and one of the growing vortices is centered about the mathematics of the schools. A most characteristic report has just been issued as a result of the Curtis tests in the New York public schools. Dr. Curtis was engaged by the committee on school inquiry to test the arithmetic abilities of the New York public school children, and he found that, compared with children of schools in other cities, they were slightly better in speed, worse in accuracy and very poor in reasoning. The problems were simple ones in addition, subtraction, multiplication and division. 33,500 students were examined in 903 classes of 52 schools. " So far as any individual child is concerned, to say that he has completed the course in arithmetic in the public schools is to convey no information as to his ability in even the simplest work. He may be almost an absolute incompetent so far as practical work is concerned or he may have acquired a degree of skill that would be adequate for any situation in which he is likely to find himself. . . . It seems probable," says the expert, " that a very simple practical course of study in arithmetic, based directly upon the social needs of the children, would influence for good a greater number of both teachers and children than any other change that could be as easily made." The Norwich tests which were conducted by Professor Terrill, in 1909, showed that the child of today could answer the questions of 1862 with greater satisfaction. My idea is not to test the student by any one form of examination at any one time but rather to institute a series of tests which shall always have a definite predetermined aim in accordance with some of the elements of our classified educational achievements. A comprehensive examination should be a spectrum of the individual's mind. I believe the time is ripe for the entrance of a great educational reformer and I hope that when he arrives we may know him as none other than an ideal teacher who has made the comprehensive examination a living reality. Our problem can not be answered at this meeting alone because it requires the concerted action of everyone working in the interests of education. It is a great national problem for upon it rests the security and integrity of American citizenship. Let each one of us return to work with a fresh impetus and with a wholesome desire to examine the examinations that we have set in the past and then reset these questions with the larger aims that have been proposed in the discussions of the comprehensive examinations. We will be vindicated by this extra effort and will get a more intimate acquaintance with the student mind. Above all other considerations we will discern that if we draw upon the student's command of resources, comprehensive vision, critical acumen, and constructive ability then we shall be honored by that greatest of appellatives,—a teacher.

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